The Wilson Effective Kähler Potential For Supersymmetric Nonlinear Sigma Models

T.E. Clark and S.T. Love
Department of Physics
Purdue University
West Lafayette, IN 47907-1396

Abstract

Renormalization group methods are used to determine the evolution of the low energy Wilson effective action for supersymmetric nonlinear sigma models in four dimensions. For the case of supersymmetric CP^{N-1} models, the Kähler potential is determined exactly and is shown to exhibit a nontrivial ultraviolet fixed point in addition to a trivial infrared fixed point. The strong coupling behavior of the theory suggests the possible existence of additional relevant operators or nonperturbative degrees of freedom.

The action, Γ, for four dimensional supersymmetric nonlinear sigma models is determined by the Kähler potential, $K(\phi, \bar{\phi})$ [1], and takes the form

$$\Gamma = \int dV K(\phi, \bar{\phi}). \tag{1}$$

Such models naturally emerge in the analysis of many underlying supersymmetric gauge theories when describing the physics below a scale Λ where the gauge nonsinglet degrees of freedom are confined but yet above the supersymmetry breaking scale. In that case, $(\bar{\phi}^{\bar{i}})$ ϕ^i denote the relevant light gauge singlet (anti-) chiral superfield degrees of freedom. Alternatively, $(\bar{\phi}^{\bar{i}})$ ϕ^i could be the Nambu-Goldstone superfields resulting from the spontaneous breakdown of an internal symmetry group G to an unbroken subgroup H at a scale Λ, at which the supersymmetry is still unbroken [2]. In either case, the effective action containing all terms through two space-time derivatives is the supersymmetric nonlinear sigma model [3]-[9] action of Eq. (1).

The self radiative corrections to the action can be determined by functionally integrating over the degrees of freedom below the scale Λ [10][11]. Thus for supersymmetric nonlinear sigma models, the Wilson effective Kähler potential at scale $\Lambda(t) = e^{-t}\Lambda, \quad t > 0$ is obtained by integrating out the degrees of freedom between Λ and $e^{-t}\Lambda$. The resulting effective action at this lower scale can be equivalently used to describe the physics on all lower energy scales. While the non-renormalization theorem [12][13] guarantees the absence of induced superpotential terms, the effective action will, in general, contain corrections to the Kähler potential as well as terms containing higher powers of space-time derivatives. These higher derivative terms will be consistently neglected in the subsequent analysis. Although this is a truncation of the model, it is an improvement over the often used local approximation [14][15] which completely neglects the radiative corrections to all terms containing derivatives. The reason we are able to go beyond the local approximation in the present case is a direct consequence of the supersymmetry which allows all terms containing up to two space-time derivatives to be derivable from a potential function [16]. Thus our analysis allows a determination of the anomalous dimension for the (anti-) chiral superfield.

Integrating out the degrees of freedom in an infinitesimal momentum shell just below the scale $e^{-t}\Lambda$ while rescaling all dimensionful parameters by $e^{-t}\Lambda$ and all fields according to their anomalous dimensionality, the compensating change in the Kähler potential can be characterized by a nonlinear partial

differential equation in t and the superfields which holds independent of the strength of the coupling. The solution to this Wilson (exact) renormalization group equation is tantamount to explicitly performing the functional integration into the infrared.

For definiteness, we consider the action of Eq. (1) for the particular case of the supersymmetric CP^{N-1} model. Here there are $N-1$ (anti-) chiral superfields, $(\bar{\phi}^{\bar{i}})\phi^i$, $i, \bar{i} = 1, ..., N-1$, whose lowest components are the coordinates for the homogeneous Kähler manifold $\frac{SU(N)}{SU(N-1)\times U(1)}$. Defining the Kähler metric at scale $\Lambda(t)$ as

$$g_{i\bar{i}} \equiv K_{i\bar{i}}(\phi, \bar{\phi}) \ , \tag{2}$$

where the subscripts on K denote differentiation with respect to the superfields, e.g. $K_{\bar{i}} = \frac{\partial K}{\partial \bar{\phi}^i}$, the Wilson renormalization group equation takes the form

$$\begin{aligned} \frac{\partial g_{i\bar{i}}}{\partial t} &= -2\gamma g_{i\bar{i}} - (1+\gamma)\phi^j g_{i\bar{i}j} - (1+\gamma)\bar{\phi}^{\bar{j}} g_{i\bar{i}\bar{j}} \\ &\quad + \frac{1}{8\pi^2}\left[g_{i\bar{i}j\bar{j}} g^{-1}_{\bar{j}j} - g_{i\bar{j}j} g_{k\bar{i}\bar{k}} g^{-1}_{\bar{k}j} g^{-1}_{\bar{j}k} \right] , \end{aligned} \tag{3}$$

where γ denotes the anomalous dimension for the $N-1$ chiral superfields. Since the fields are rescaled according to their anomalous dimensions as the system flows into the infrared, the chiral field anomalous dimension can be extracted by evaluating equation (3) at $\phi^i = 0 = \bar{\phi}^{\bar{i}}$ where $g_{i\bar{i}}|_{\phi^i = 0 = \bar{\phi}^i} = \delta_{\bar{i}i}$. So doing, one finds that

$$2\gamma \delta_{i\bar{i}} = \frac{1}{8\pi^2}\left[g_{i\bar{i}j\bar{j}} g^{-1}_{\bar{j}j} - g_{i\bar{j}j} g_{k\bar{i}\bar{k}} g^{-1}_{\bar{k}j} g^{-1}_{\bar{j}k} \right] |_{\phi^i = 0 = \bar{\phi}^i} \ . \tag{4}$$

Assuming that the Kähler potential is a function of the product of chiral and antichiral superfields $\rho \equiv \bar{\phi}^{\bar{i}} \delta_{\bar{i}i} \phi^i$ with $\delta_{ii} = N-1$, the Wilson equation for the effective Kähler potential, $K(\rho, t)$, then reduces to

$$\frac{\partial K}{\partial t} = 2K - 2(1+\gamma)\rho K_\rho + \frac{1}{8\pi^2}\left[\ln\left(K_\rho + \rho K_{\rho\rho}\right) + (N-2)\ln K_\rho \right] \ , \tag{5}$$

while the anomalous dimension of the chiral superfields is given by

$$\gamma = \frac{N}{16\pi^2} K_{\rho\rho}|_{\rho=0}. \tag{6}$$

Note that the $\rho = 0$ normalization of the metric translates into the normalization $K_\rho|_{\rho=0} = 1$. The partial differential equation (5) describes the exact renormalization group flow of the effective action for any pure chiral theory where the quantum radiative corrections have been truncated to include only the Kähler potential which are the terms that are at most of order p^2 in its momentum expansion.

Contrary to most Wilson renormalization group equations which can only be treated by numerical means, Eq. (5) admits the analytical solution

$$K(\rho, t) = \frac{1}{\chi(t)} \ln\left(1 + \chi(t)\rho\right) \tag{7}$$

where $\chi(t)$ is an effective coupling constant satisfying the renormalization group equation

$$\frac{d\chi(t)}{dt} = -2(1+\gamma)\chi(t), \tag{8}$$

and where the anomalous dimension is

$$\gamma(t) = -\frac{N}{16\pi^2}\chi(t). \tag{9}$$

The form of the solution displayed in Eq. (7) is not altogether unanticipated. For models with fields in homogeneous Kähler manifolds, the form of the Kähler potential is uniquely determined by the group structure of the G/H coset space. The Wilson renormalization group flow of the Kähler potential must then describe the exact renormalization group running of the finite number of coupling constants (decay constants) in such models. For the case of the supersymmetric CP^{N-1} model, there is one such coupling and the Kähler potential in terms of a particular set of coordinates can be written in the form given by Eq. (7).

Defining the scaled coupling constant $\kappa(t)$ as

$$\kappa(t) \equiv \frac{N}{16\pi^2}\chi(t), \tag{10}$$

the renormalization group equation becomes

$$\frac{d\kappa}{dt} = -2\kappa(1-\kappa) \tag{11}$$

while the anomalous dimension is simply

$$\gamma(t) = -\kappa(t) \; . \tag{12}$$

Thus, within the higher derivative operator truncation we are working, the exact beta function for κ takes the simple form

$$\beta_\kappa = 2\kappa(1-\kappa) \tag{13}$$

and is seen to be a sum of two terms only. We reiterate that in obtaining this result we have included contributions from an infinite number of operators. A similar form for the beta function has previously been advocated [17] based on extrapolating the results of a $(2+\epsilon)$ expansion calculation[18][19]. For two dimensional supersymmetric sigma models such a behavior for the beta function has also been previously argued [20][21] using an alternate chain of reasoning and demonstrated explicitly by calculation through four-loop order [22].

The beta function displays an ultraviolet fixed point at $\kappa = \kappa_c \equiv 1$, in addition to the trivial infrared fixed point. The renormalization group equation is readily integrated producing the explicit solution

$$\kappa(t) = \frac{\kappa(0)}{(1-\kappa(0))e^{2t} + \kappa(0)}. \tag{14}$$

Alternatively expressed, the ultraviolet fixed point is indicative of the existence of a phase trasition with an associated renormalization group invariant inverse correlation length [17] $\xi^{-1} = \Lambda(\frac{1}{\kappa(0)} - \frac{1}{\kappa_c})^{\frac{1}{2}}$ satisfying

$$\xi^{-1} = \Lambda(t)\left(\frac{1}{\kappa(t)} - \frac{1}{\kappa_c}\right)^{\nu'} , \tag{15}$$

where $\nu' = \frac{1}{2}$ is a critical exponent.

Figure 1 illustrates the various renormalization group flows. For negative bare couplings, $\kappa(0) < 0$, the theory evolves from a Landau singularity in the ultraviolet towards the trivial fixed point in the infrared. Note that these flows admit a simple particle interpretation with a free field theory emerging in the far IR. On the other hand, the flows for positive bare coupling either approach the trivial fixed point if the bare coupling is less than the UV

Figure 1: Renormalization group flow of the coupling constant κ. The arrows denote the coupling constant evolution as the system flows into the infrared. An ultraviolet non-trivial fixed point ocurrs at $\kappa = \kappa_c \equiv 1$.

fixed point, $\kappa(0) < \kappa_c = 1$, or run away if $\kappa(0) > \kappa_c = 1$. However, for each of these flows, the scalar superfield propagator goes as $\frac{\Lambda(t)^{-2\gamma}}{p^{2-2\gamma}}$ with a negative anomalous dimension, $\gamma < 0$. This signals a violation of unitarity. In order for this pathological behavior to be circumvented, the model must either contain additional relevant or marginal operators or admit additional nonperturbative degrees of freedom whose presence will reverse the sign of the anomalous dimension for positive bare coupling. The fact that higher derivative operators may not be irrelevant in the context of $2+\epsilon$ expansions for sigma models has been discussed in [23][24][25] and could have some bearing on this issue. Indeed it is just this class of operators which we have neglected in our current analysis. Whether the relevance of these operators is simply an artifact of the $2+\epsilon$ expansion or could be invalidated by higher order calculation is still an open question. In addition, it is known that in lower dimensions the CP^{N-1} model asdmits nontrivial classical solutions; instantons in d=2 and monopoles in d=3. Thus one might speculate that

the lack of a simple particle interpretation for positive bare coupling for the d=4 supersymmetric CP^{N-1} model is reflective of the need to include string degrees of freedom.

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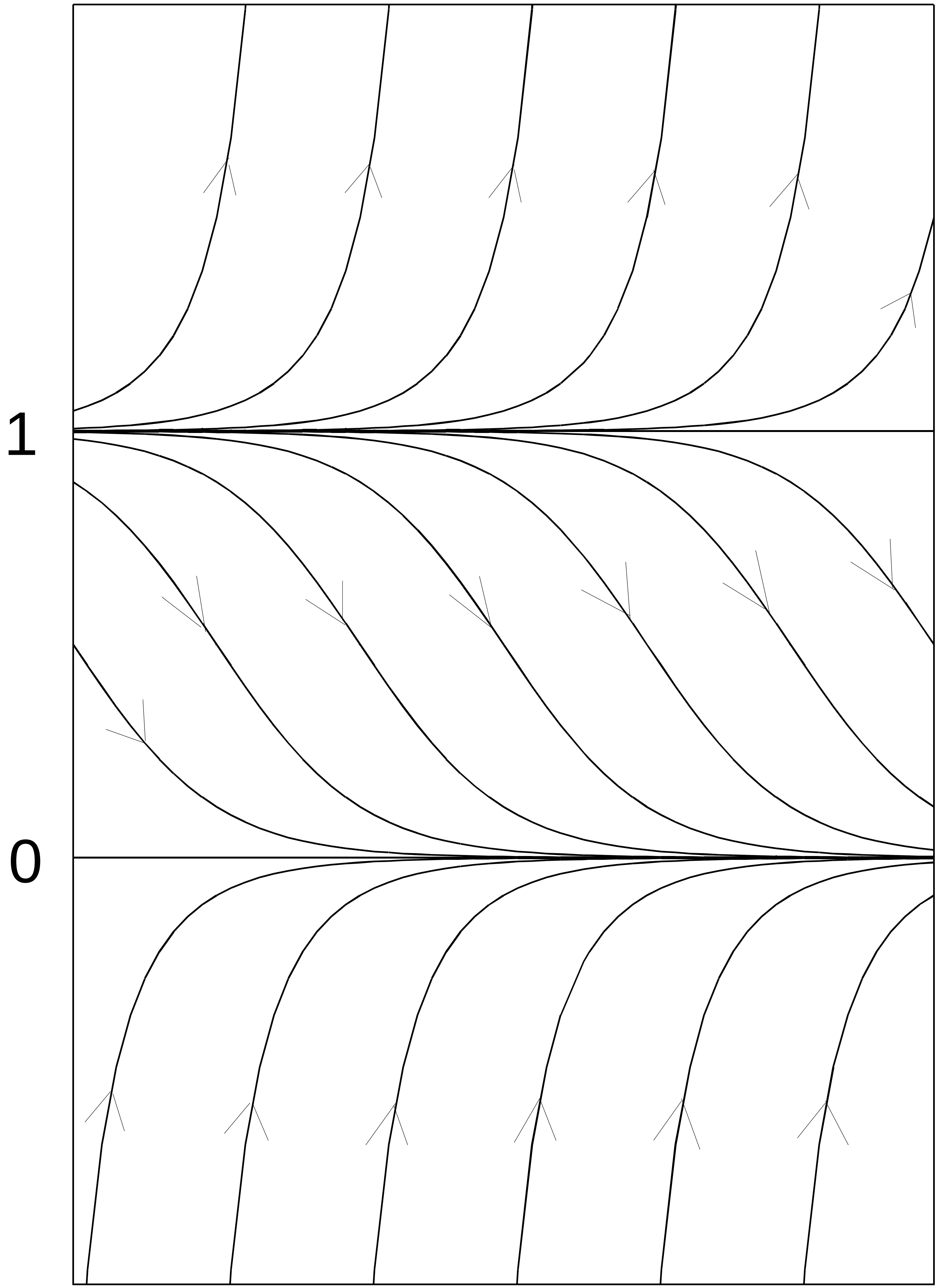
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